

# **The Role of Fracture Toughness in the Mechanics of Scratching Surfaces of Ductile Materials: Implications for Abrasion, Wear and Grinding**

by

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## Abstract

It is well known that when the surface of a ductile solid is scratched by a pointed tool, the resulting groove is formed by one of two distinct modes of deformation, depending on the inclination of the point to the surface (Sedriks & Mulhearn, 1963). When the point is drawn almost parallel to the surface (at low tool ‘attack’ angles), the material displaced from the groove becomes piled up in ridges alongside the scratch by a process of plastic flow. At greater attack angles, the displaced material is removed in ribbon form by a process of cutting with no pile-up.

A recent analysis (Atkins & Liu, 2006) shows that the critical tool attack angle, at which the transition takes place between the two modes, depends upon the fracture toughness of the material being scratched as well as yield strength, friction and depth of groove. The depth of groove during sliding under deadweight loading is different from the depth resulting from initial indentation: at very small attack angles, the depth can be smaller than the static indentation depth, but as the attack angle increases, so does the depth of groove formed by cutting.

The relevance of the model to abrasive wear and polishing is discussed, along with implications for Krushschov-Babichev wear resistance diagrams and the Archard equation. The analysis explains why the specific energy in grinding increases at small depths of cut.

Measurement of the critical attack angle in scratching with a facet-first indenter may possibly be a way to estimate the fracture toughness of small samples when other, more conventional, methods are difficult to carry out.

A J Sedriks and T O Mulhearn, ‘Mechanics of cutting and rubbing in simulated abrasive processes’, *Wear*, **6**, 1963, 457-466.

A G Atkins and J H Liu, ‘Toughness and the transition between cutting and rubbing in abrasive contacts’, *Wear*, 2006, in press

- Material is removed from a groove formed by scratching in one of two ways, depending mainly on the inclination of the scratching point:

- (i) by cutting a chip;
- (ii) by forming a 'prow' in front of the tool through which material is displaced up into ridges alongside the groove.

In (i) material is separated from the surface and removed; in (ii) no material is removed (in theory); the process is similar to "ironing"

- Cutting mode occurs at large 'attack angles'  $\beta (= (\alpha + 90)^\circ$  where  $\alpha$  is the rake angle of metalcutting); rubbing mode occurs at low attack angles.

- Tool forces (parallel and perpendicular to the surface) are low at small  $\beta$  but rise at greater  $\beta$  only to fall again at even greater  $\beta$ .

- A theory for rubbing concerns (a) work of plastic flow in forming the prow ahead of the tool and the flow of material into the ridges alongside the groove; and (b) work of friction between tool and material.

- A theory for cutting concerns (c) work of separation of material in addition to plastic flow and friction. It is machining with a triangular (or other shaped tool). Recent work for all machining processes has shown that the specific work of separation  $R$  is at  $\text{kJ/m}^2$  levels in ductile metals (like the  $J_C$  parameter of ductile fracture mechanics). Inclusion of such specific works has resolved many unanswered problems of traditional metalcutting theories (A G Atkins "Modelling metalcutting using modern ductile fracture mechanics: quantitative explanations for some longstanding problems", *Int J Mech Sci* **45** 373-396 2003).

- Forces predicted by theory for rubbing continually *increase* with  $\beta$  to very large values; forces predicted by theory for cutting continually *decrease* as  $\beta$  increases. Where they intersect gives the tool inclination at which the transition between the two modes of deformation occurs.

- The  $\beta_{\text{critical}}$  value at transition depends upon (a) the point angle of the tool; (b) friction; and (c) the (toughness/yield strength) ratio ( $R/\tau_y$ ) of the material combined into a non-dimensional parameter  $Z = (R/\tau_y)t$  where  $t$  is the depth of the groove.

- Experimental measurement of  $\beta_{\text{critical}}$  enables  $R$  to be found knowing the hardness of a material (to which  $\tau_y$  is related) and the coefficient of friction  $\mu$  (from the scratching forces). The technique is valuable when only very small samples are available such as in archaeological investigations. The change in design of the ancient Greek Corinthian helmet before the battles of Marathon and Thermopylae was explained employing these properties (P H Blyth and A G Atkins "Stabbing of metal sheets by a triangular knife: an archaeological investigation" *Int J Impact Engr* **27** 459-473 2002).

Table 1

Material	$\mu$ (all +/- 5°)	$\beta_{\text{critical}}$ (MPa)	$\tau_y$ mm	Z	t mm	$R/\tau_y$ kJ/m <sup>2</sup>	R
Lead	0.6(5)	55	7.5	4	0.17	0.6(8)	51
Aluminium	1.2	85	90	25	0.05	1.2	100
Copper	0.4	45	200	1	0.04(7)	0.05	9
$\alpha$ -brass	0.5(5)	55	340	4	0.03(2)	0.12	43
nickel	0.7	65	530	5	0.02(6)	0.13	69

- Alternatively, toughness and yield strength can be determined simultaneously from cutting tests in which the forces are measured at different depths of cut. Cutting theory gives

$$F_H = (1/Q)[\tau_y \gamma]wt + (1/Q)Rw \quad \text{and} \quad F_V = F_H \tan(\beta - \alpha)$$

where  $F_H$  is the force parallel to the surface and  $F_V$  that perpendicular to the surface.  $\beta$  is the friction angle ( $\tan \beta = \mu$ ) and  $\alpha$  is the tool rake angle. The width of cut is  $w$  and the uncut chip thickness is  $t$ .  $Q$  is a friction factor involving  $\beta$ ,  $\phi$  and  $\alpha$  where  $\phi$  is the inclination of the primary shear plane. The shear strain on the primary shear plane is  $\gamma$  and theory shows that  $\gamma$  is constant above a limiting  $t$  (actually below a limiting  $Z$ ) since  $\phi$  is then constant [a significant contribution of the new theory is to demonstrate that  $\phi$  values are material dependent, through  $Z$ , which is well known experimentally].

It is seen that yield strength depends on the *slope* of a plot of cutting force versus depth of cut; toughness is obtained from the force *intercept* at zero depth of cut. Results should pass through the origin according to traditional ‘plasticity and friction’ theories. Intercepts are found in practice but are explained away in terms of rubbing on the clearance face of the tool. The new theory says that an intercept is to be *expected*.

- The expression for  $F_H$  may be re-written

$$F_H/wt = [\tau_y \gamma] + R/t$$

to give the so-called ‘specific cutting pressure’, ‘specific energy’ or ‘unit power’. It is well known that this parameter increases markedly at small depths of cut. Traditional theories have no explanation, but the new theory predicts a large increase in  $(F_H/wt)$  not only because of the obvious inverse  $(R/t)$  term but also because  $\gamma$  increases at small  $t$ .

- Abrasive wear, processes like grinding and so on, all involve scratching surfaces at very small depths of cut. Why the specific energy in grinding is so large (and produces high interfacial contact temperatures) is clear from the equation above.

- Why abrasive papers and grinding wheels are relatively inefficient is because material is removed only when grooves are formed by cutting (ie at relatively large attack angle  $\beta$ ) and in practice the number of grits having such favourable orientations is limited. Most grits produce prows and ridges. It is true that ridges can be knocked off in subsequent passes but, overall, the process is not efficient.
- Were all material removed from the grooves removed as debris, and were the depth of grooves determined solely by hardness, wear rate would depend directly on the load and inversely on hardness (the Archard equation). But the depth of grooves formed in sliding depends on the (toughness/yield strength  $R/\tau_y$ ) ratio of the material of the material. The wear resistance of very hard solids is not as great as expected on the basis of the wear resistance of a softer solid, because the toughness of very hard solids is relatively smaller and the ( $R/\tau_y$ ) ratio is reduced. Reduced ( $R/\tau_y$ ) results in greater groove depths at the same load at all attack angles, so the wear resistance (being inversely proportional to  $t^2$ ) must decrease relatively. This is what Krushschov & Babichev found many years ago (“On the relation between the hardness of metals and their wear resistance to rubbing against an abrasive surface, Vestnik Mashinostroeniya **34/9** 3-9 1954).
- Different thermomechanical treatments of alloys will alter the ( $R/\tau_y$ ) ratio in different ways. It should not be surprising therefore that harder metals can sometimes be cut with smaller forces than softer versions of the same alloy. Traditional cutting theories say that hard materials require greater cutting forces than soft materials.
- Similar considerations will apply to erosion (e.g. M Papini and S Dhar “Experimental verification of a model of erosion due to the impact of rigid single angular particles on fully-plastic targets” Int J Mech Sci **48** 469-482 2006, where earlier references will be found).
- The width and depth of grooves (on surfaces of damaged vehicles, say) can be used forensically to establish the likely loads required to produce such grooves. This permits judgement to be made on whether those loads are normal in-service loads or the result of fault or accident conditions.
- To Conclude:
  - (i) the mechanics of groove formation by the two usual modes have been presented
  - (ii) the transition between cutting and rubbing has been explained in terms of lesser work or force
  - (iii) the role of fracture toughness and the importance of the toughness/strength ratio ratio ( $R/\tau_y$ ) of the scratched material has been highlighted
  - (iv) the importance of the non-dimensional parameter  $Z = (R/\tau_y)t$  in controlling cutting by a pointed tool has been emphasised
  - (v) the implications of the analyses for a number of practical applications has been investigated.