Coupling Mechanics of Antikythera Gearwheels

F. Sorge
DICGIM,
University of Palermo,
Viale delle Scienze,
90128 Palermo, Italy
e-mail: francesco.sorge@unipa.it

This paper discusses the gear coupling mechanics of the ancient Antikythera mechanism, among whose distinctive characteristics was the triangular shaping of the teeth. The engagement of the tooth pairs is analyzed in detail, estimating the temporal variation of the speed ratio due to the back and forth shifting of the relative instant center. The admissibility of the theoretical contact points is carefully checked, and the magnitude of the successive tooth collisions is calculated together with the energy losses arising from the particular nature of the coupling. Some interesting results are that only one tooth pair turns out to be active at each time instant and the real path may belong only to the approach or to the recess region entirely, or may split into separate subphases, in approach and in recess, or may even straddle both regions. The occurrence of each of these conditions depends on the average speed ratio (tooth ratio) and the assigned clearance between the wheels. It is also found that the speed oscillation is roughly contained in a ±10% range and the efficiency may reach rather high values, despite the presumable crude finishing of the ancient gearwheels due to the rather rudimentary technology used in the construction of the tooth profiles. [DOI: 10.1115/1.4006530]

1 Introduction

Though relatively few residues from the antiquity machinery are still preserved in some museums scattered over the world, a fairly advanced and diffused construction technology had been achieved and an extended practical use of many mechanical devices had grown up, especially in the Hellenistic, Byzantine, and Islamic worlds. Originally, the machine manufacturing of ancient times developed more from contingent practical requirements than from a planning skill grounded on precise scientific conceptions. Following this heuristic approach to machine design, the first treatises on mechanics where mathematical principles were systematically applied appeared after the fourth century B.C. (Ctesibius, Philo, Hero of Alexandria, and Archimedes of Syracuse). The main interest was generally on the war machines, such as ballistae, catapults, burning glasses, steam cannons, giant claws, etc., as this type of research was generously sponsored by the despots of the time. For example, extended mathematical searches were addressed to the problem of the cube-root, whose solution was fundamental for dimensioning the cables of the catapult torsion springs (see the introductory chapters of the books [1–2] and Refs. [3–5]).

The gearwheel coupling was no doubt a rather current application and, for example, was largely used for the implementation of astronomical devices like planetary calculators for the position of celestial bodies, astrolabes, or odometers.

One of the most significant find, the Antikythera mechanism (Fig. 1), is a planetary gear system used for the description of several astronomical trajectories, which presumably dates from the first century B.C. It was firstly ascribed to the astronomer Hypparchus of the Academy of Rhodes, which was founded by the philosopher Posidonius, but a new hypothesis transfers its construction to some colony of Corinth. It was retrieved at the beginning of the 20th century from the Antikythera wreck, after its accidental discovery, thanks to some sponge-divers anchored near the coast of the homonymous island (*'Αντικύθηρα*, whose meaning is "in front of Kythera," is a very small Greek island with less than100 inhabitants located half-way in the sea channel between Crete and the larger island of Kythera: see Fig. 1(*b*)).

Many in-depth studies have been carried out on its functionality as a primitive analog computer (e.g., see Refs. [6–11] and mind the recent activity and many papers of the Antikythera Mechanism Research Project [12]). Pastore [6] reports an extensive description of this gear system and elucidates its functional characteristics. De Solla Price spent a lot of time in his studies about this mechanism in order to reconstruct the missing parts starting from the few archeological residuals, and also tried to assemble a complete model, whose copy is now in the National Archaeological Museum of Athens. Wright and Bromley carried out a wide campaign of X-ray tomographic detection of the wheels, continued later on by the Antikythera Mechanism Research Project, which pointed out the equilateral triangular shape of the toothing unequivocally. Wright and Bromley reconstruction of the planetary system corrects the previous model of Price, conjecturing also the presence of some epicyclic mechanism (Fig. 1(*c*)).

On the other hand, it is sensational that such triangular profiles reveal a less advanced design conception in comparison with the recent find of the gearwheel of Olbia (Sardinia, Italy), which may be ascribed to Archimedes of Syracuse (third century B.C.) and is then earlier than the Antikythera planetary of more than one century. This fragment exhibits the extraordinary characteristic that the tooth profiles are very close to the modern cycloidal shape, suggesting that Archimedes, whose complete works have been saved till today to a very limited extent, may have been perhaps involved with the study of the cycloid curves [6].

It is very probable that many gear systems like the Antikythera mechanism were built during the Hellenistic period. Cicero mentions two other devices of this type in *De Re Publica* and says that they had been built by Archimedes and one of them was brought to Rome by Marcus Claudius Marcellus, who conquered Syracuse during the Second Punic War. It is supposed that the cycloidal gearwheel of Olbia belonged to one of these devices [6]. Cicero also says that other similar devices had been built "recently" and thus, this technology was quite spread since the time of Archimedes and the Antikythera orrery was just one exemplar of a widely diffused manufacturing, though skilled hands and complex calculations were necessary for this type of construction.

The complex technology needed for the construction of these gear systems was transmitted, through the Hellenistic culture, to the Byzantine and Islamic worlds and gave the conceptual origin to several geared machines in the Middle Age, like odometers, mechanical calendars, and clockworks (for example, bear in mind some machines described in the books of the Byzantine monasteries, the volume Kitab al-Hiyal by Banu Musa, the odometers and astrolabes of al Biruni, etc.).

Contributed by the Mechanisms and Robotics Committee of ASME for publication in the Journal of Mechanical Design. Manuscript received September 8, 2011; final manuscript received March 22, 2012; published online April 27, 2012. Assoc. Editor: Hong-Sen Yan.

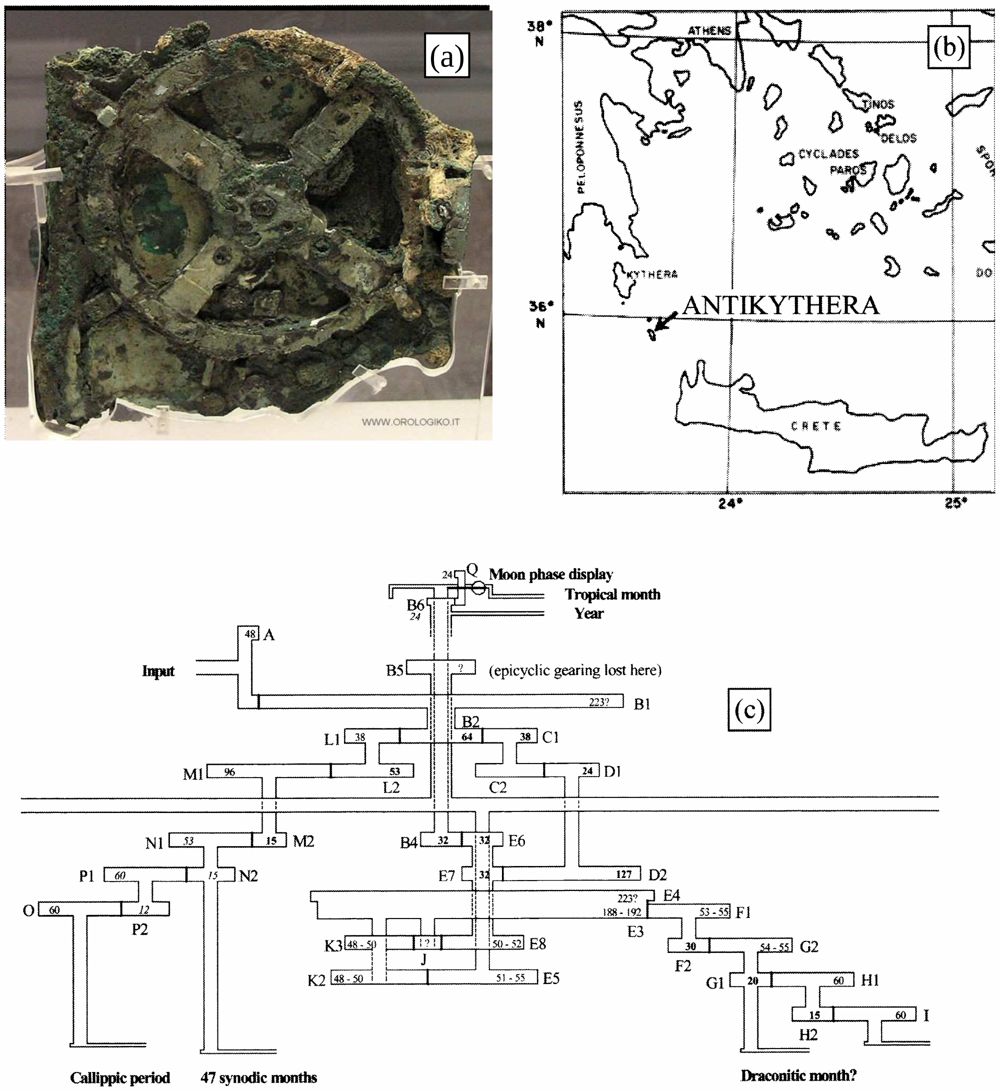

Fig. 1 (*a*) Main fragment of the Antikythera mechanism at the National Archaeological Museum of Athens. (*b*) Site of the Antikythera wreck. (*c*) Presumed scheme of the whole planetary mechanism (Ref. [10]).

We can also find gear devices in the codices of Leonardo da Vinci, i.e., at the dawn of the Modern Era. However, the tooth shape is still rudimentary, often triangular or trapezoidal, and far from the modern cycloidal or involute profiles, whose common application developed quite after Euler during the Industrial Revolution [13] (it is to be remarked that the involute profiles are ascribable to Euler, whereas the cycloid profiles were conceived a long time before, probably by Girard Desargues or Philippe de La Hire in the 17th century).

The present analysis, far from addressing all unresolved questions of historical kind concerning the Antikythera mechanism, resumes and completes the work of Sorge [14] with the mean aim at stating the fundamentals of the mechanical coupling of the Antikythera triangular gears. There seems to be no previous study of this type.

2 Geometry of Triangular Toothed Gears

A scheme of the gear coupling is represented in Figs. 2(*a*)–2(*c*) for three possible contact configurations: approach, profile matching, and recess. The sizes are chosen in accordance to the design concepts to be presumed for those ancient times. Some elementary constraints lead to forced choices of the system data (constant pitch, equal tooth depth, etc.), while other variables are to be assumed ad lib. The tooth numbers are chosen within the range of experimental observations [6–11]. Several tooth ratios are applied to the model in order to encompass a wide area of real verisimilar conditions.

The ratio of the tooth numbers of the two gears, z_1 of the driver wheel and z_2 of the driven one, may be considered as an average speed ratio imposed by the machine operation, but the simple geometrical shape of the tooth profiles implies the variability of the instant speed ratio during the meshing.

It is firstly supposed that each pair of conjugate teeth engages along the whole ideal path, ignoring the possible restrictions involved by the presence of the preceding and following teeth. In actual fact, the number of active teeth and the true line of contact for sequential tooth gearings are limited by the need of avoiding interference conditions and will be examined in Secs. 3.3 and 3.4.

It is assumed that the teeth on the one and the other gear have the same aperture angle 2β (the experimental results on the Antikythera gearwheels indicate that $\beta = 30$ deg) and the same depth h from the vertex V to the root circle (see Fig. 3), whence, fixing the tooth numbers, the whole toothing can be designed relying on evident geometrical considerations.

Given β and z, the ratio of the tip radius R to the root radius $R-h$ turns out to be fixed (applying the law of sines, we know now that this ratio is given by $\sin(\beta+\pi/z)/\sin\beta$). The triangle OBV of Fig. 3 shows that the tooth depth h and the width e of the side profile are calculable as

have been compensated in part by the lubrication. Thus, energy losses of the order of 10% or less may be reasonably conjectured.

References

[1] Hartenberg, R. S., and Denavit, J., 1964, *Kinematic Synthesis of Linkages*, McGraw-Hill, New York.

[2] Dimarogonas, A. D., 2001, *Machine Design. A CAD Approach*, Wiley & Sons, New York.

[3] Chondros, T. G., 2007, "Archimedes (287–212 B.C.)," *History of Mechanism and Machine Science, 1, Distinguished Figures in Mechanism and Machine Science, Their Contribution and Legacies*, M. Ceccarelli, ed., Springer, The Netherlands, ISBN 978-1-4020-6365-7, pp. 1–30.

[4] Chondros, T. G., 2008, "The Development of Machine Design as a Science From Classical Times to Modern Era," HMM International Symposium on History of Machines and Mechanisms, Tainan, Taiwan, publ. by Springer, The Netherlands, ISBN 987-1-4020-9484-2, pp. 59–68.

[5] Rossi, C., 2010, "Archimedes Cannons Against the Roman Fleet?," *History of Mechanism and Machine Science, 11*, The Genius of Archimedes. 23 Centuries of Influence on Mathematics, Science and Engineering, M. Ceccarelli, ed., Springer, The Netherlands, ISBN 978-90-481-9091-1, pp. 113–131.

[6] Pastore, G., 2010, *Il Planetario di Archimede Ritrovato* (The Recovered Archimedes Planetarium), © Copyright 2006–2012–Giovanni Pastore, www.giovannipastore.it, Roma, ISBN 978-88-904715-2-0, in Italian, (directly available from the author).

[7] De Solla Price, D. J., 1959, "An Ancient Greek Computer," Sci. Am., **200**(6), pp. 60–67.

[8] De Solla Price, D. J., 1974, "Gears From the Greeks. The Antikythera Mechanism: A Calendar Computer From ca. 80 B.C.," Trans. Am. Philos. Soc., New Ser., **64**(7), pp. 1–70.

[9] Wright, M. T., Bromley, A. G., and Magkou, E., 1995, "Simple X-ray Tomography and the Antikythera Mechanism," PACT. Revue du groupe européen d'études pour les techniques physiques, chimiques, biologiques et mathématiques appliquées à l'archéologie (J. Eur. Study Group Phys., Chem., Biol. Math. Tech. Appl. Archaeol.), **45**, pp. 531–543.

[10] Wright, M. T., 2005, "The Antikythera Mechanism: A New Gearing Scheme," Bull. Sci. Instrum. Soc., **85**, pp. 2–7.

[11] Wright, M. T., 2006, *The Greek Planetarium: A New Reconstruction of the Antikythera Mechanism* (AIA Lectures Program), Imperial College, London, UK.

[12] The Antikythera Mechanism Research Project, 2007, March 23, www.antikythera-mechanism.gr.

[13] Sorge, F., 2011, "On the Gear Mechanics Before the Cycloid and Involute Profiling," XX AIMETA Congress, Bologna, Italy, Sept. 12–15.

[14] Sorge, F., 2011, "A Note on the Mechanics of Ancient Gear Systems," 13th World Congress in Mechanism and Machine Science IFToMM, Guanajuato, México, June 19–25.